

Superconductivity and Mottness: Exact Results

Nature Physics, vol.16, 1175-1180 (2020)
with N&V by J. Zaanen

arxiv.org/abs/2103.03256

Luke Yeo



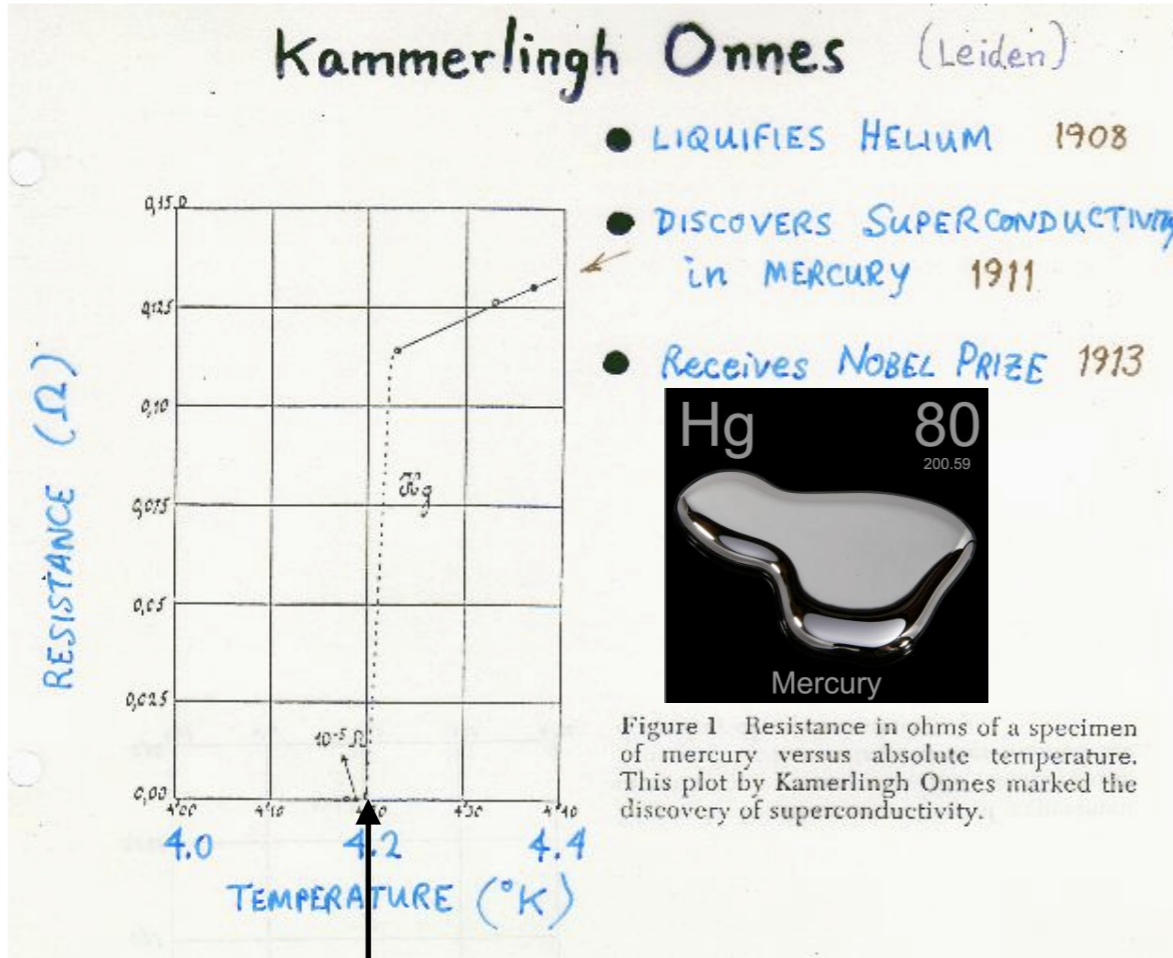
Edwin Huang



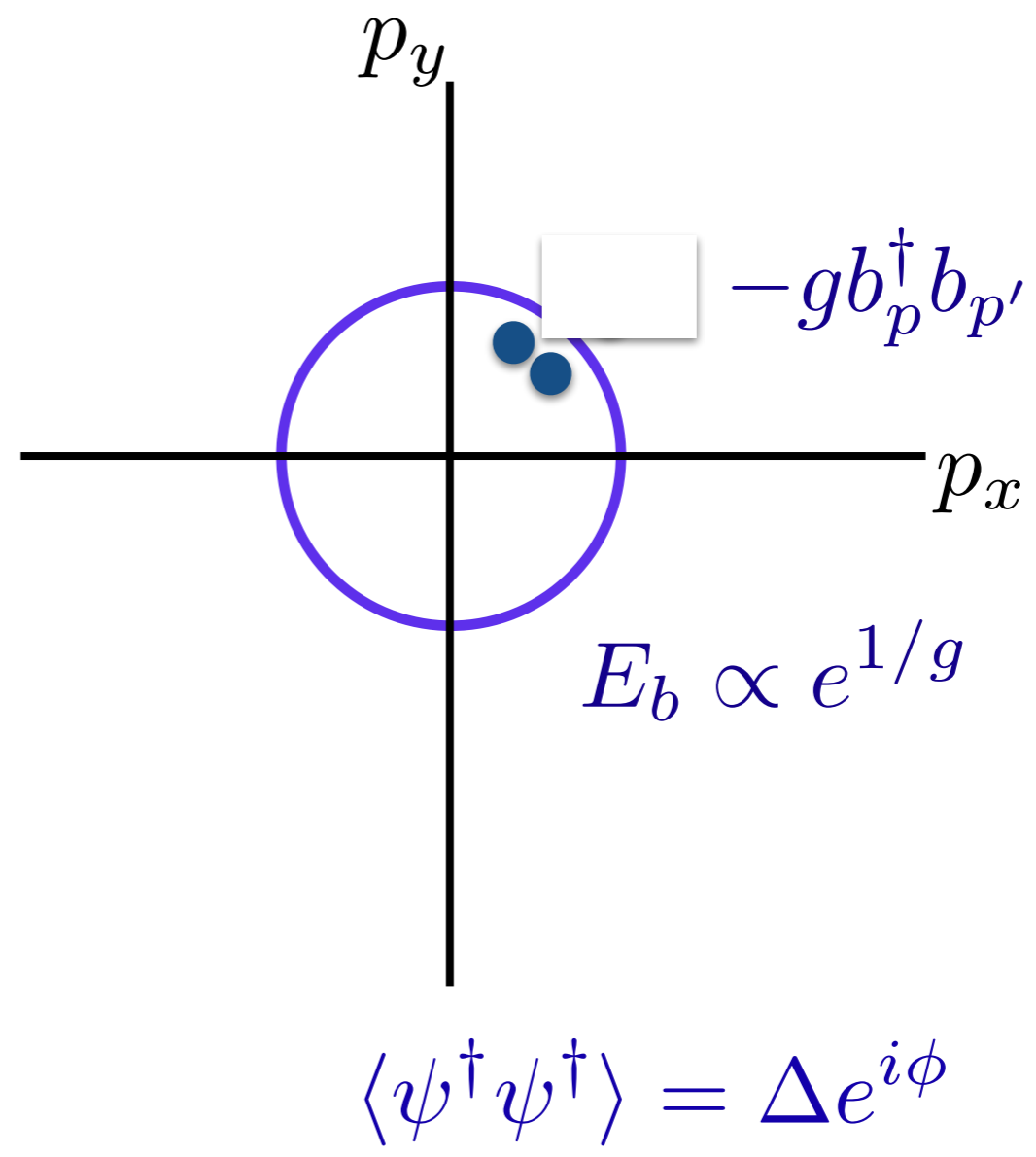
G. La Nave



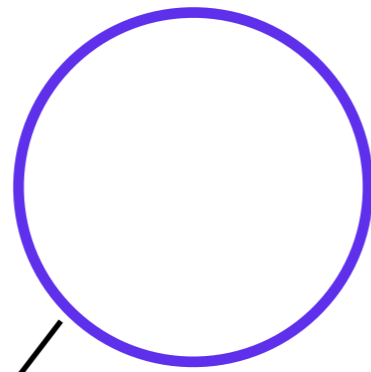
Cooper instability



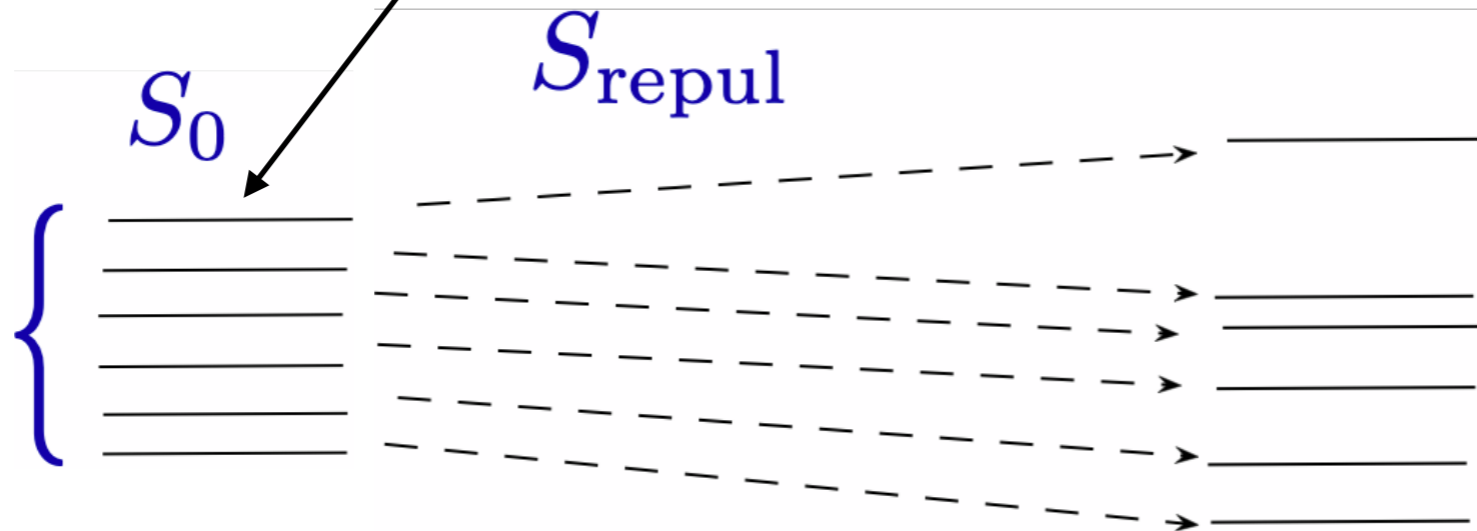
T_c



FL \rightarrow BCS



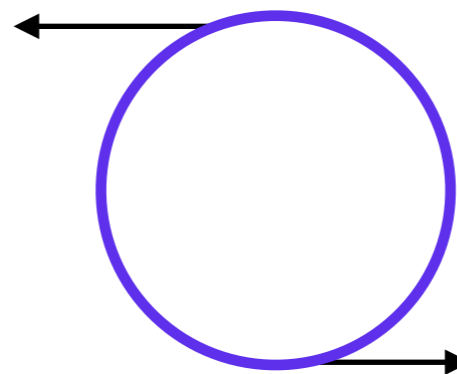
Fermi Surface



1-1
correspondence

$$[S_{\text{repul}}] > 0$$

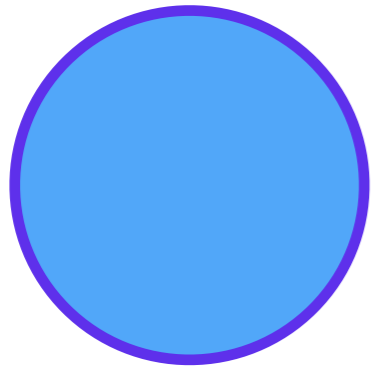
S_{repul}
irrelevant



$+S_{\text{pair}}$

superconductivity $\frac{2\Delta}{T_c} = 3.5$

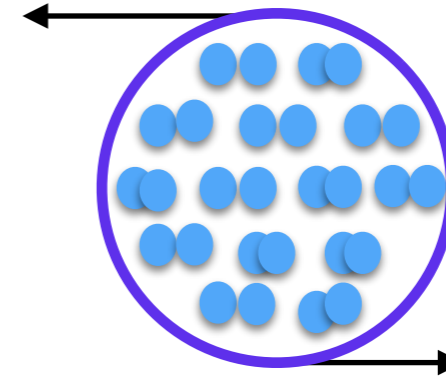
Fermi gas



Fermi liquid

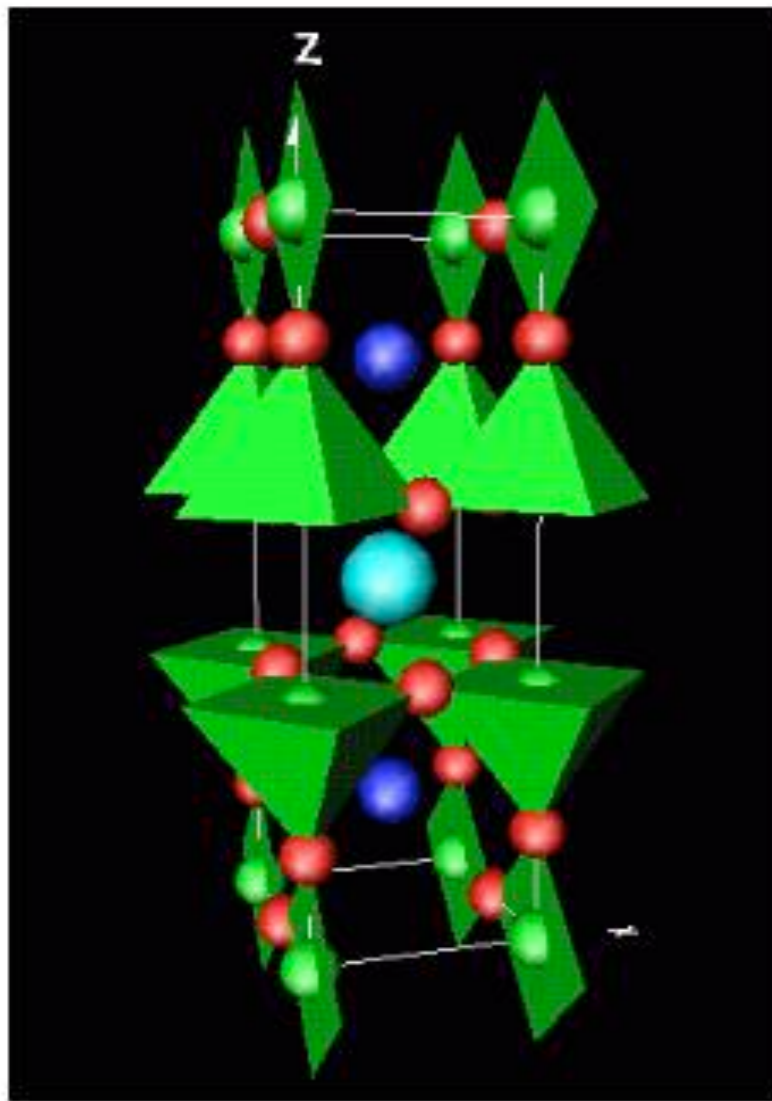


BCS
superconductor



Is there physics
beyond
BCS?

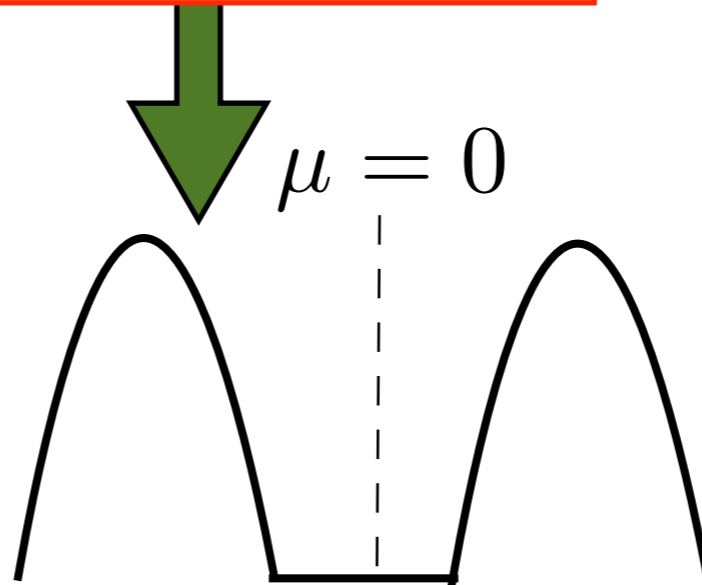
fixed
point beyond
FL?



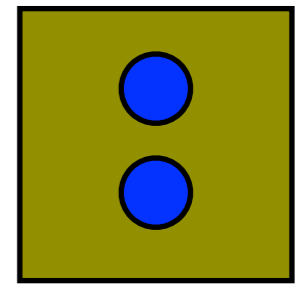
$\text{YBa}_2\text{Cu}_3\text{O}_7$
Cuprate Superconductors



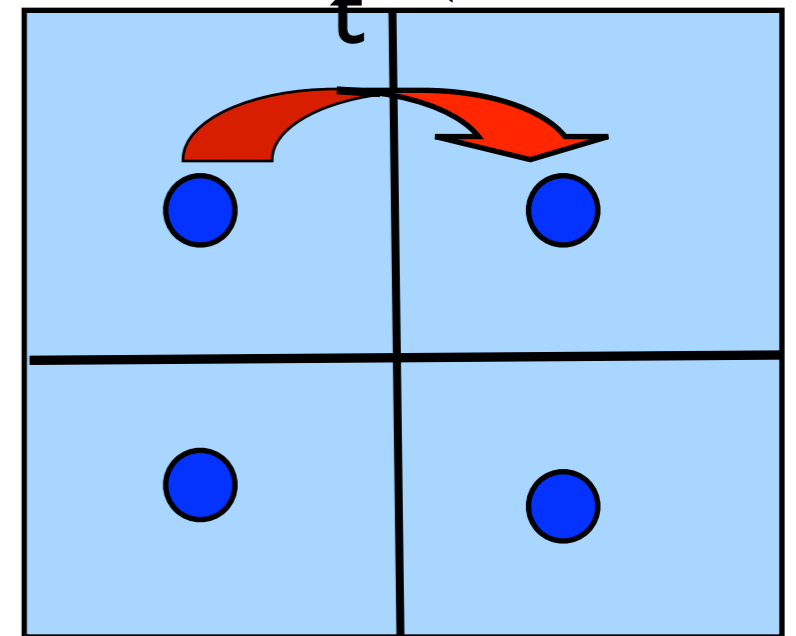
local real-space physics



NiO insulates d^8 ?
perhaps this costs energy



$$U \gg t$$



no change in size of Brillouin zone

solve the Hubbard Model!!

Cooper instability??

Progress thus far?

DMFT

QMC

disputes

Sept. 1997

A Critique of Two Metals

R. B. Laughlin
Department of Physics
Stanford University
Stanford, California 94305

idea is either missing or improperly understood. Another indicator that something is deeply wrong is the inability of anyone to describe the elementary excitation spectrum of the Mott insulator precisely even as pure phenomenology. Nowhere can one find a quantitative band structure of the elementary particle whose spectrum becomes gapped. Nowhere can one find precise information about the particle whose gapless spectrum causes the paramagnetism. Nowhere can one find information about the interactions among these particles or of their potential bound state spectroscopies. Nowhere can one find precise definitions of Mott insulator terminology. The upper and lower Hubbard bands, for example, are vague analogues of the valence and conduction bands of a semiconductor, except that they coexist and mix with soft magnetic excitations no one knows how to describe very well.

Nov. 1997

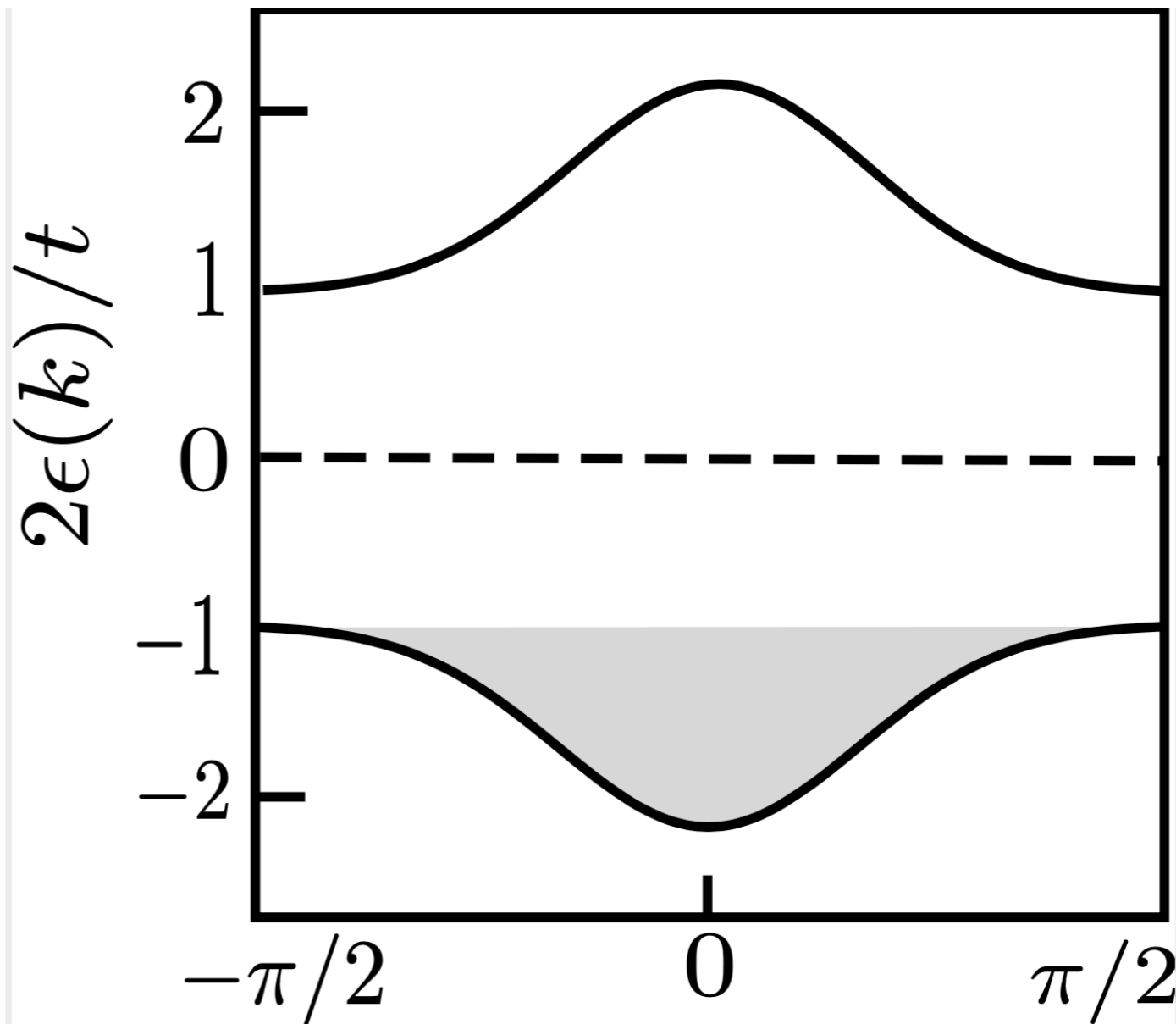
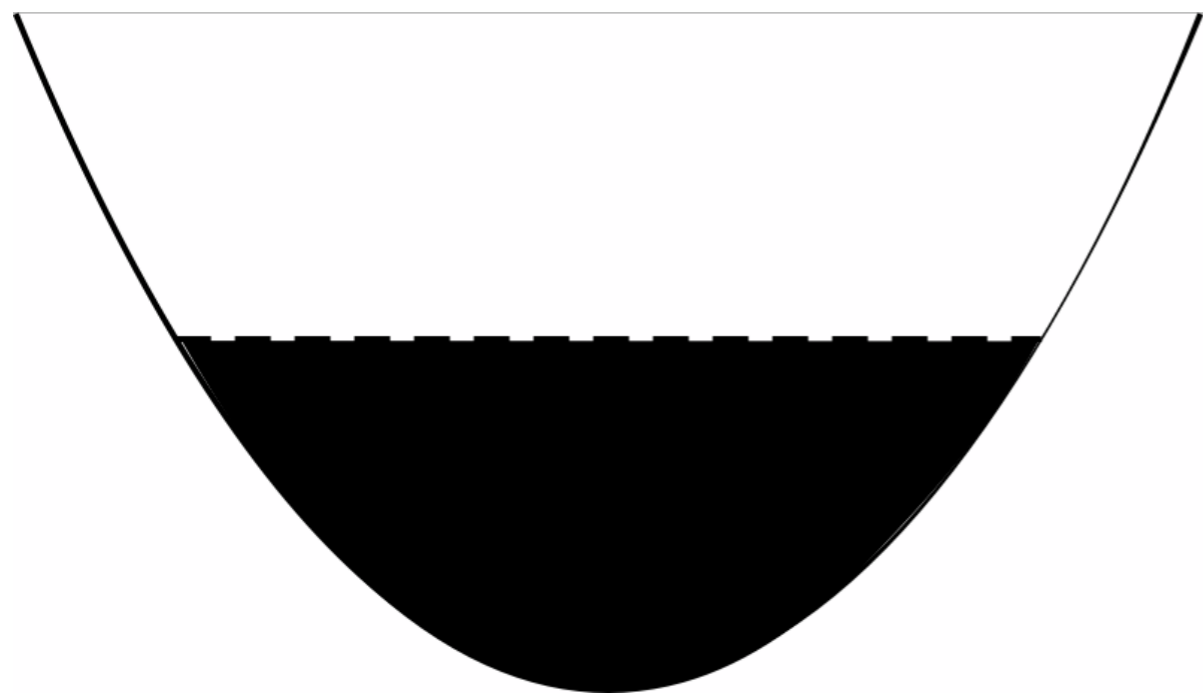
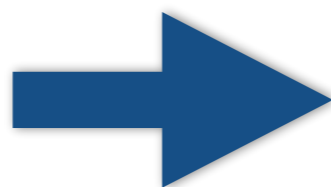
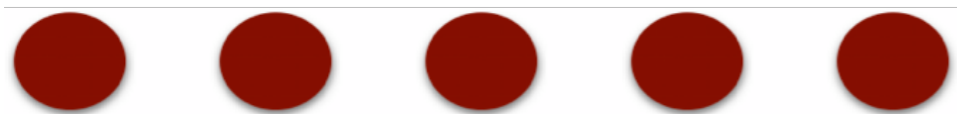
A Critique of “A Critique of Two Metals”

Philip W. Anderson and G. Baskaran

Joseph Henry Laboratories of Physics

Princeton University, Princeton, NJ 08544

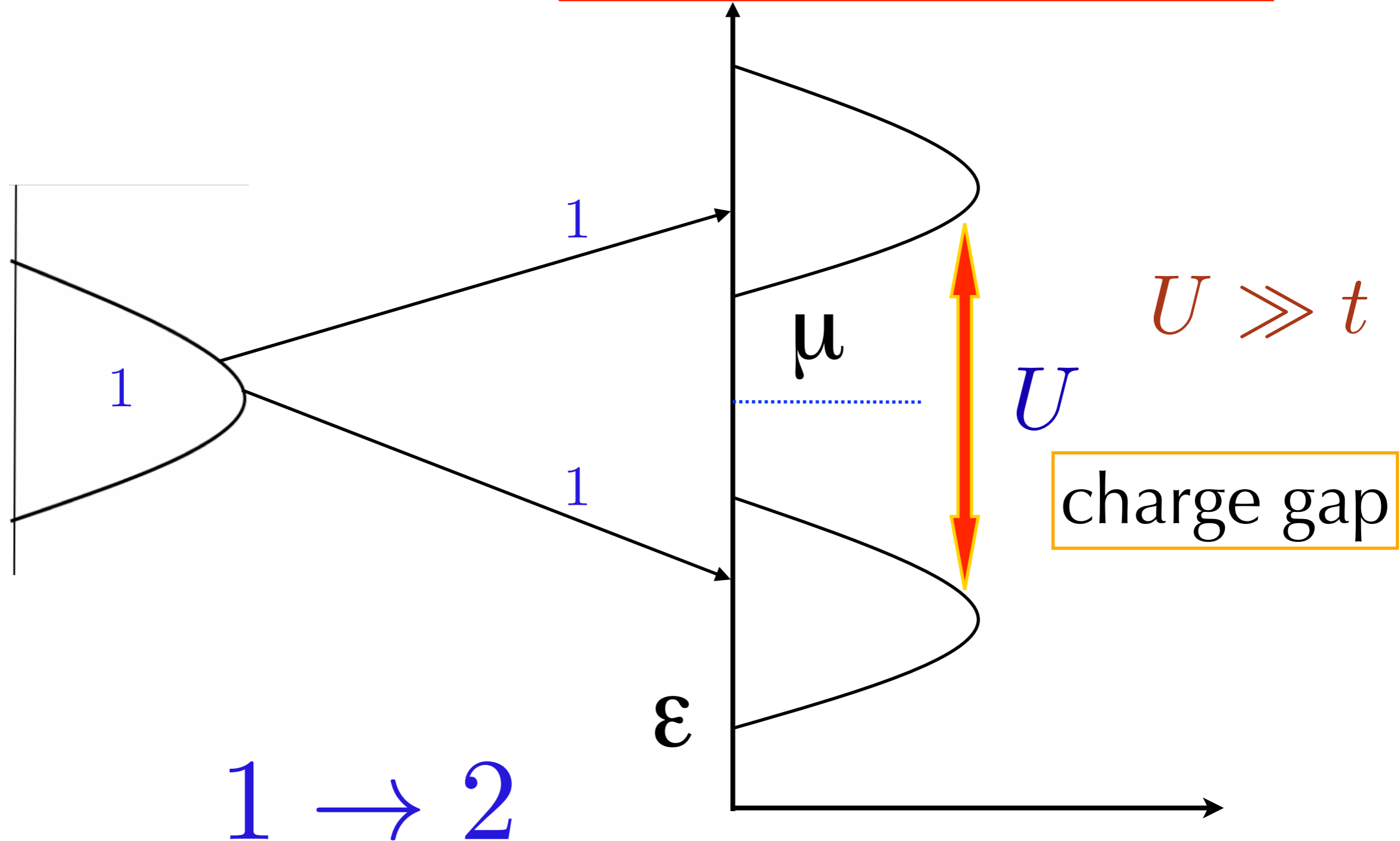
The fundamental argument is presented in the second paragraph: “Ten years of work by some of the best minds in theoretical physics have failed to produce any formal demonstration”...of the Mott insulating state. The statement would be ludicrous if it were not so influential. The proviso “at zero temperature” is added, because of course most Mott concern. It is the tragedy of Mott that although he almost certainly won his Nobel prize for the Mott insulator, Slater, who couldn’t think clearly about finite temperature, won the publicity battle.



No Mott Problem

Laughlin's objection:

gap with no symmetry breaking not demonstrated!!





—



= Mottness

zeros

$= 0$ $\neq 0$
DetReG($\omega = 0, p$)

MOTTNESS

BY

TUDOR DAN STANESCU

LICEN., University of Bucharest, 1991

M.S., University of Illinois at Urbana-Champaign, 1997

Acknowledgments

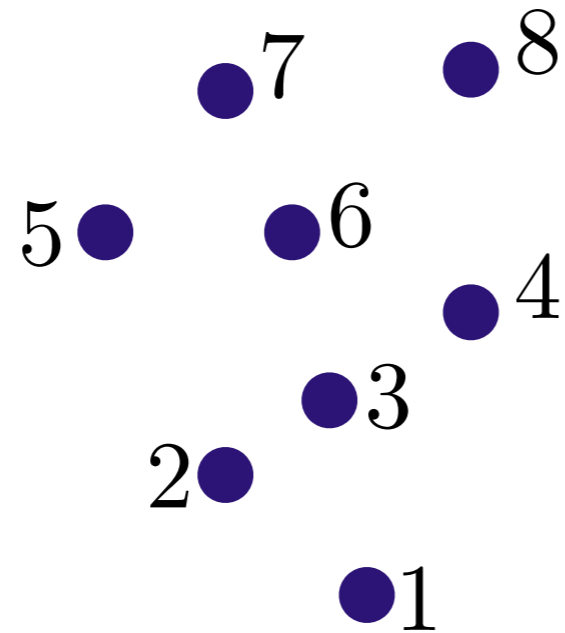
Behind the words and the formulas of this thesis are not only my thoughts, but the teaching, advice and support of many people.

First, I would like to express my gratitude to my wonderful advisor, Prof. Philip Phillips. This work was nourished by his energy, creativity and patience. From Philip I learned to look for those things that can make a difference and to never let go: when

the road seems to come to an end, it always continues around the corner.

Urbana, Illinois

counting particles



is there a more efficient way?

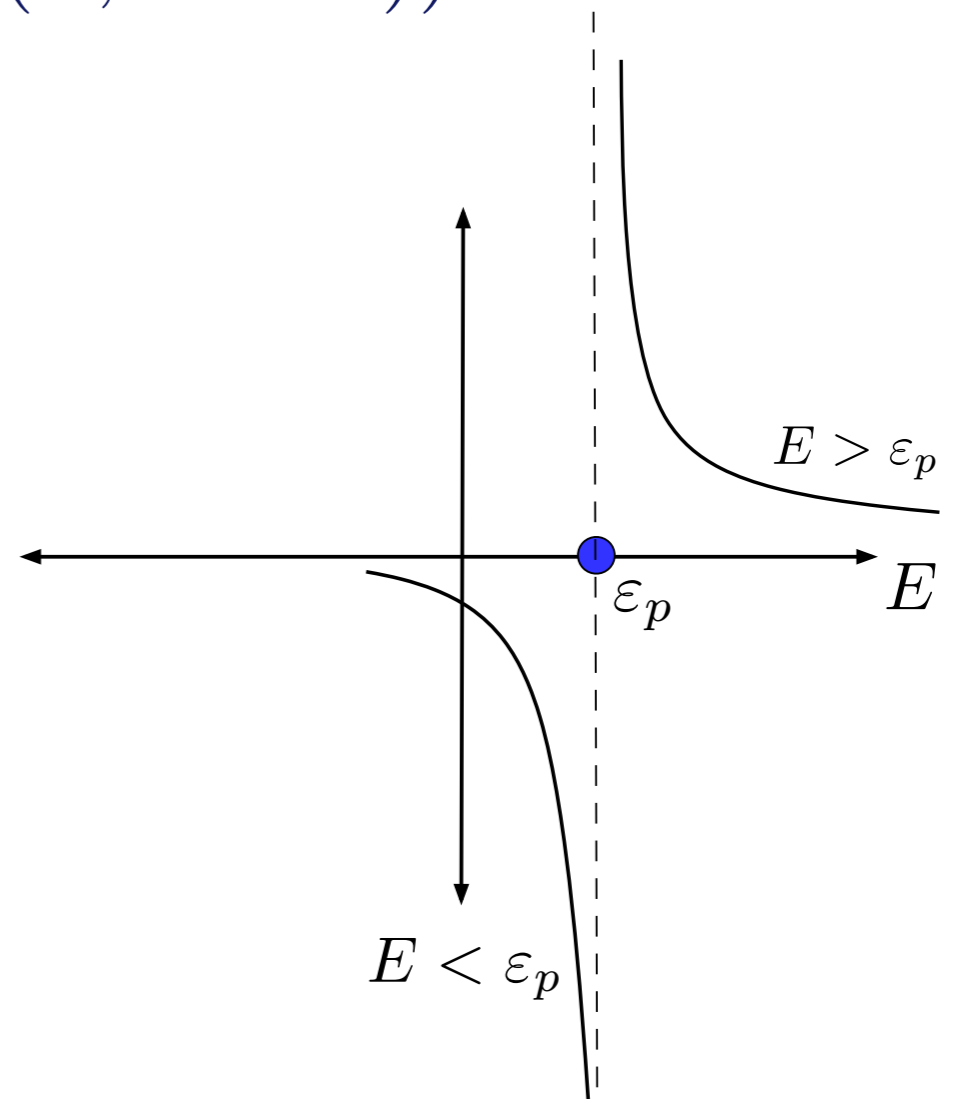
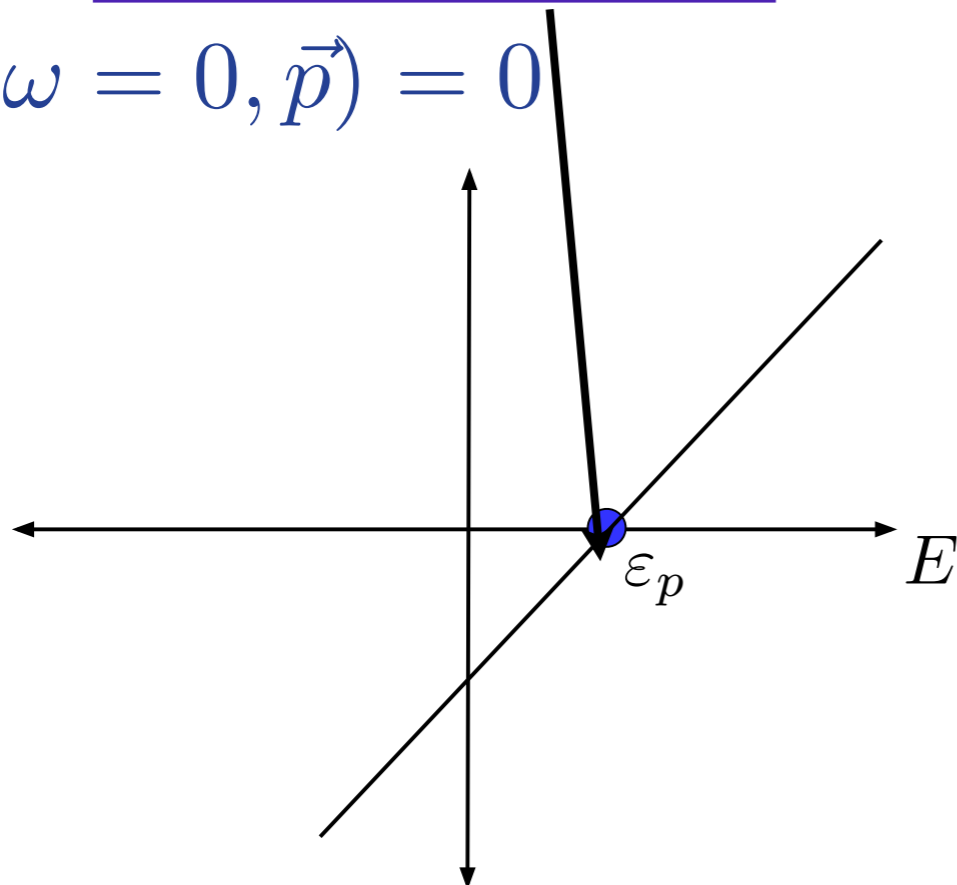
Luttinger counting theorem

$$G(E) = \frac{1}{E - \varepsilon_p}$$

$$n = 2 \sum_{\mathbf{k}} \Theta(\Re G(\mathbf{k}, \omega = \mathbf{0}))$$

zero-crossing

$$\text{Det}G(\omega = 0, \vec{p}) = 0$$



counting poles (qp)

How do zeros obtain?

$$\text{Re}G(0, p) = \int_{-\infty}^{\infty} \left(\text{band structure} \right) d\omega$$

$\mu = 0$

$\text{Im } G = 0$

Kramers-Kronig

$$= \text{below gap} + \text{above gap} = 0$$

$$\text{DetRe}G(k, \omega = 0) = 0 \text{ (single band)}$$

strongly correlated gapped systems

zeros

no propagation

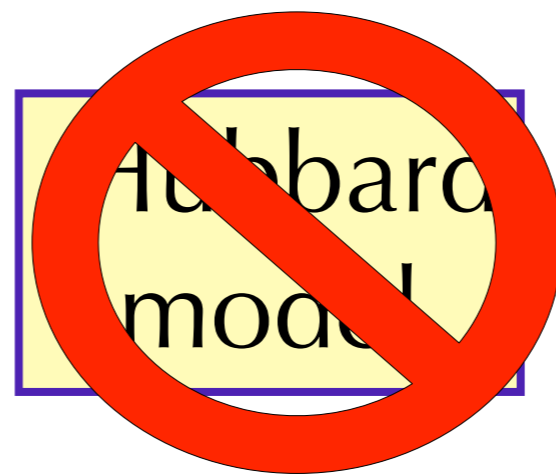


breakdown
of particle
concept

Mottness

$n = \text{zeros} + \text{poles}?$

Minimal model
for Mottness?





Anderson
Haldane
2000

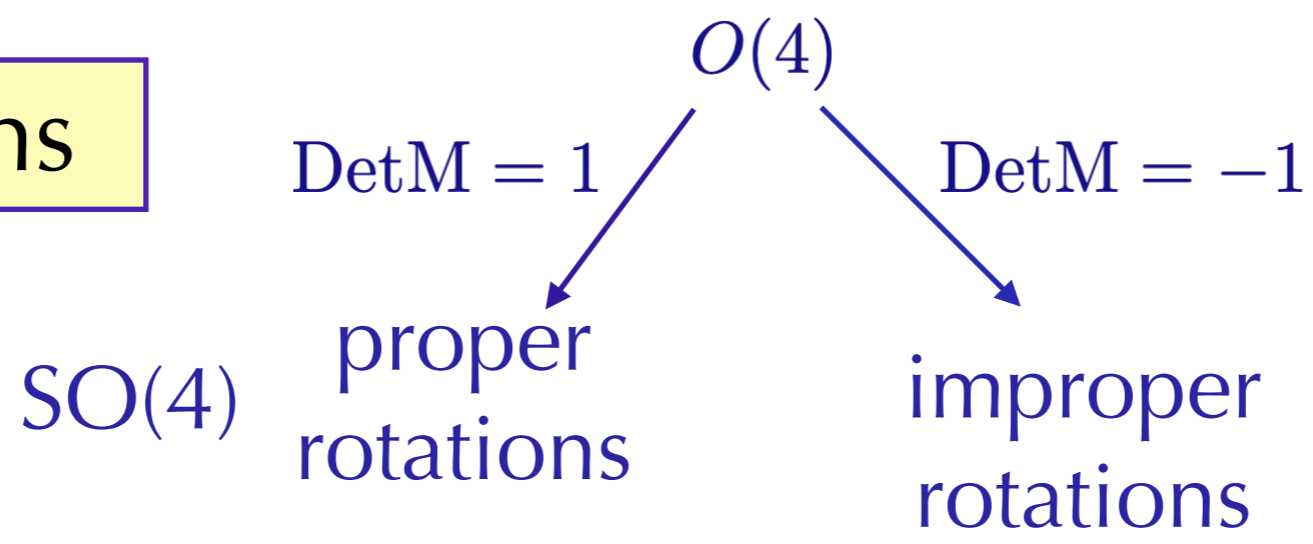
3 citations

Fermi liquids

$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + \dots \rightarrow 0$$

$(n_{p\uparrow}, n_{p\downarrow})$ conserved currents

$(c_{p\uparrow}, c_{p\downarrow}, \text{h.c.})$ 4 objects



$$\text{Det}M = \pm 1 \implies Z_2 = O(4) \div SO(4)$$

$$\epsilon(p) = \epsilon_F$$

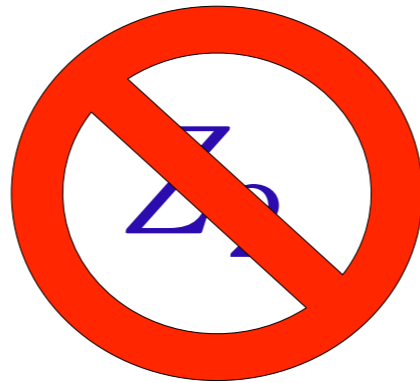
Fermi
Surface

$$H = 0$$



$$\left. \begin{array}{l} n_{p\uparrow} \rightarrow -n_{p\uparrow} \\ n_{p\downarrow} \rightarrow n_{p\downarrow} \end{array} \right\} \mathbb{Z}_2 \text{ at Fermi surface only}$$

How to destroy Fermi liquids?



$$H = \sum_{p,\sigma} (\epsilon(p) - \epsilon_F) n_{p\sigma} + U n_{p\uparrow} n_{p\downarrow}$$

odd
under Z_2

scaling dimension

$$[n_{p\uparrow} n_{p\downarrow}] = -2$$

relevant
interaction

New fixed point!

Hatsugai-Kohmoto
model

Hubbard
not
necessary!

General HK Model

$$\sum_k (\xi_k (n_{k\uparrow} + n_{k\downarrow}) + U n_{k\uparrow} n_{k\downarrow})$$

relevant
perturbation

$$[H_t, H_U] = 0$$

Solvable Mott transition

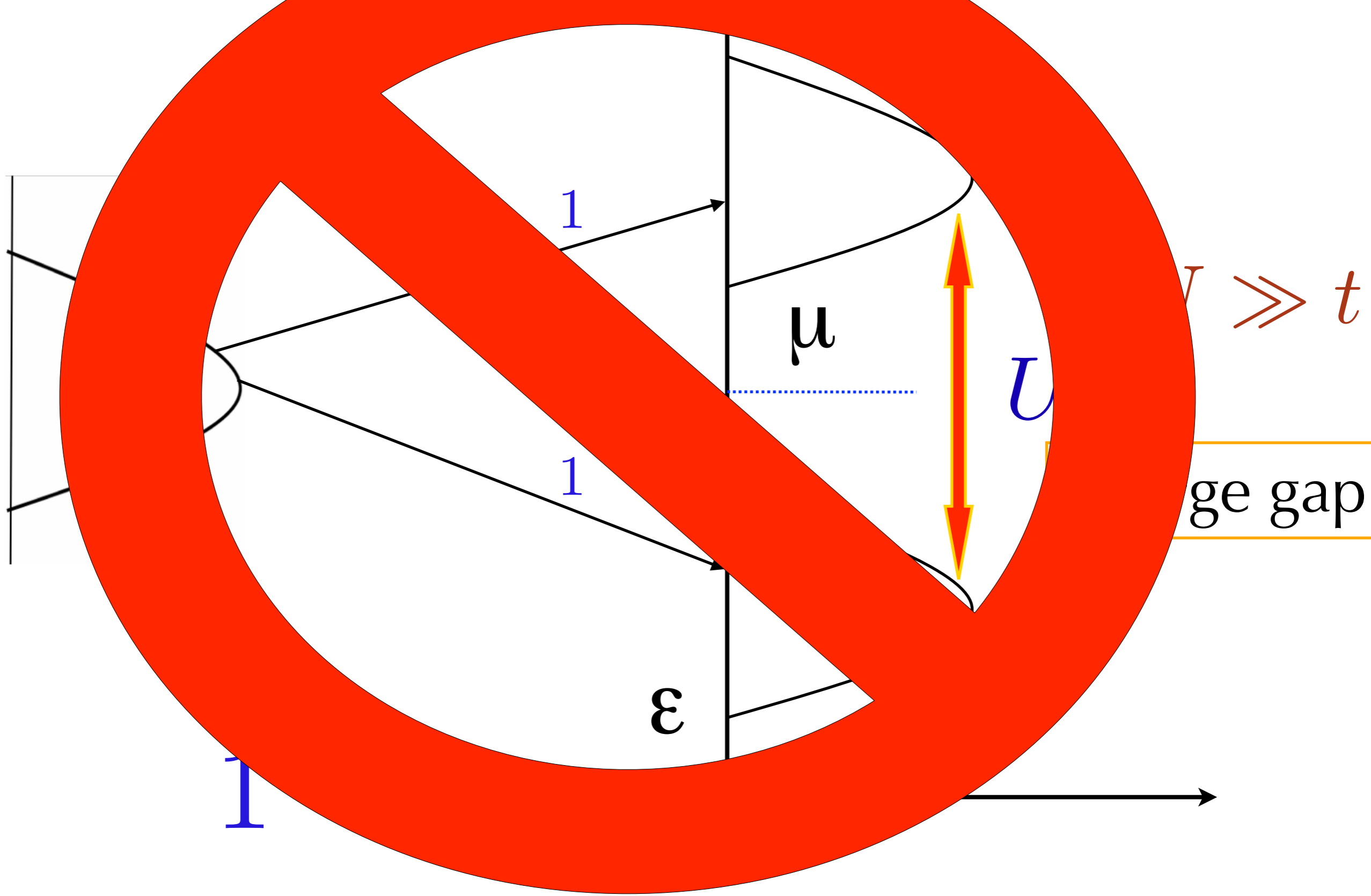
$$G_{k\sigma}(i\omega_n \rightarrow z) = \frac{1 - \langle n_{k\bar{\sigma}} \rangle}{z - \xi_k} + \frac{\langle n_{k\bar{\sigma}} \rangle}{z - (\xi_k + U)} \neq \frac{1}{z - \xi_k}$$

lower Hubbard band

upper Hubbard band

Laughlin's objection

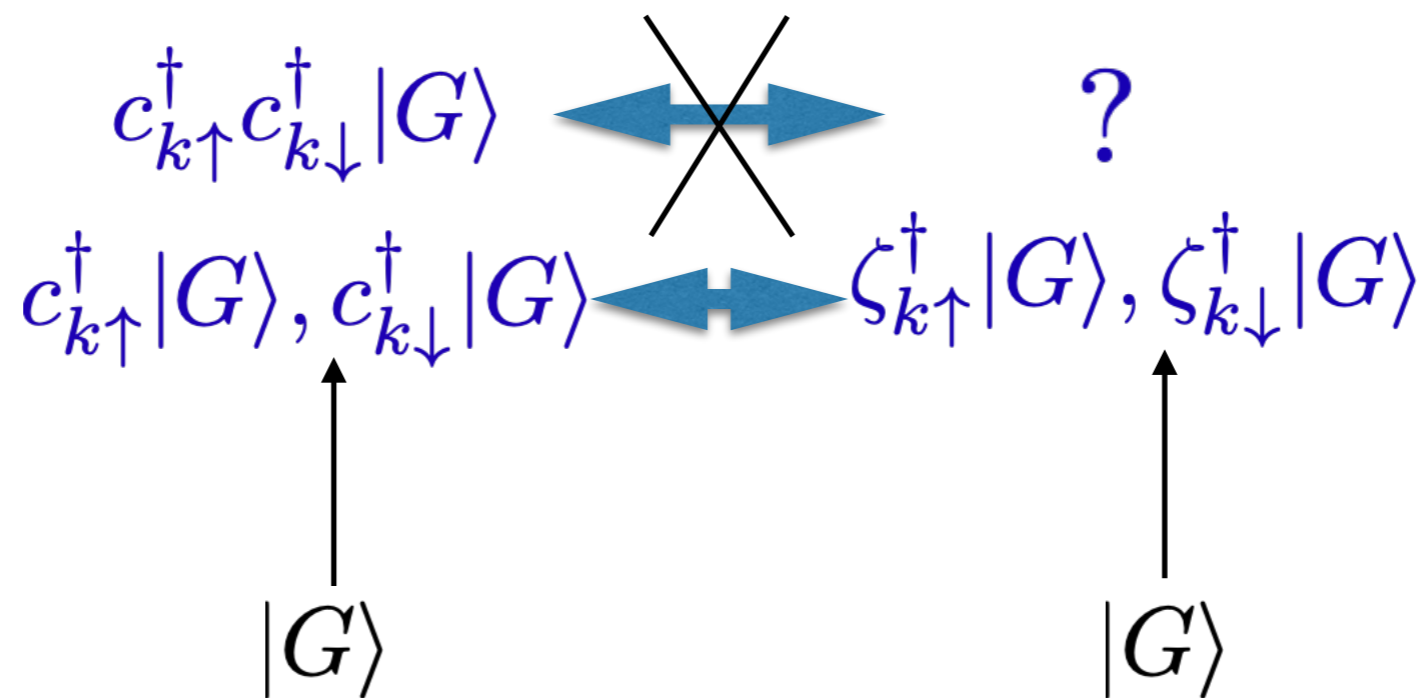
gap with no symmetry not demonstrated!!



Why NFL?

FL

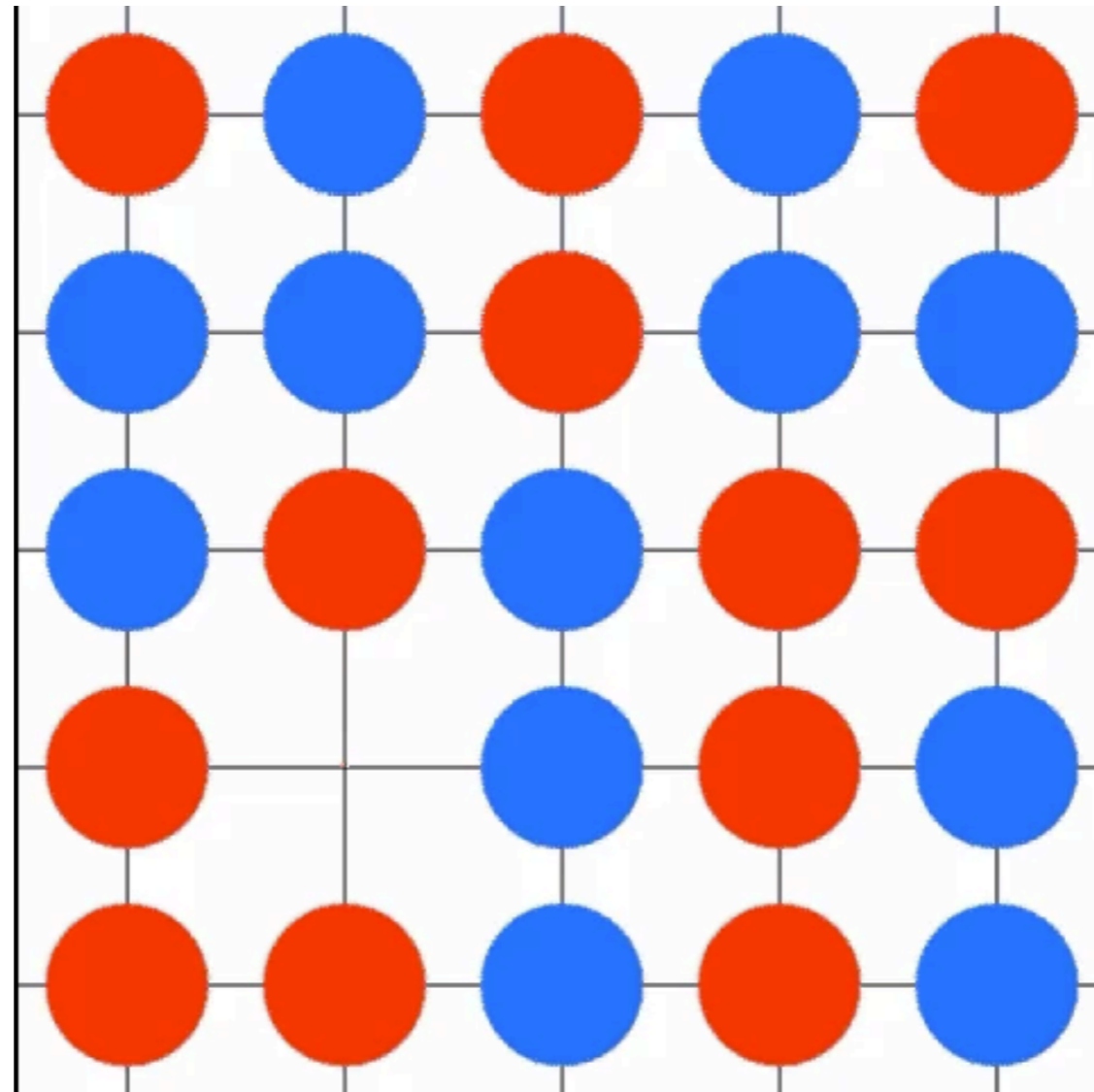
HK
Mottness



$$\zeta_{k\uparrow}^\dagger \zeta_{k\downarrow}^\dagger |G\rangle = 0$$

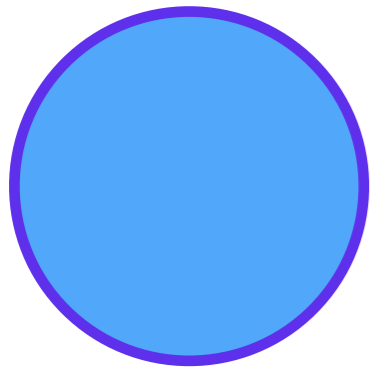
what does the HK model leave out??

$$[H_t, H_U] \neq 0$$

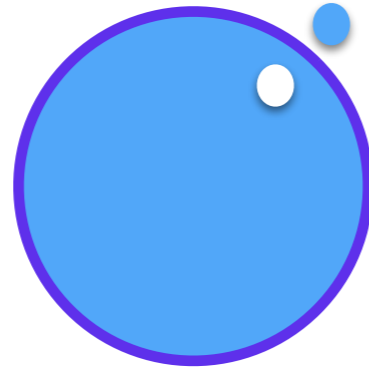


dynamical spectral weight transfer

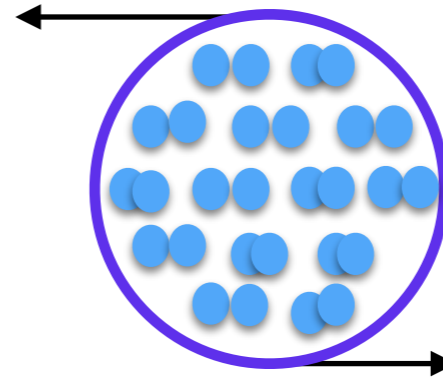
Fermi gas



Fermi liquid



BCS
superconductor



Mottness

2

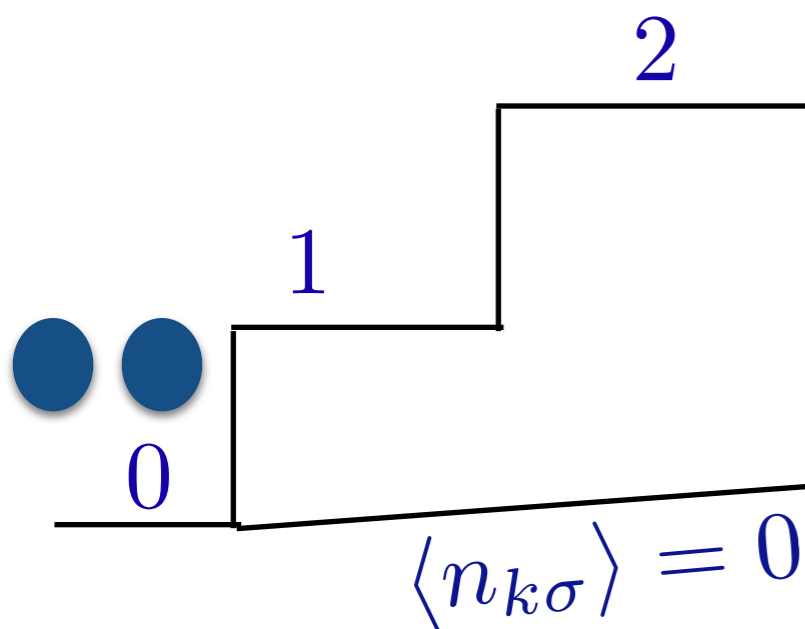
1

0



Superconductivity?

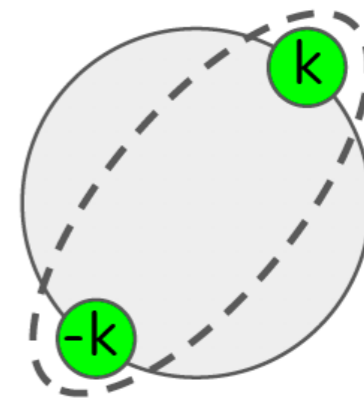
Cooper Instability



$$\langle n_{k\sigma} \rangle = 0$$

$$H = H_{\text{HK}} - gH_p$$

$$|\psi\rangle = \sum_{k \in \Omega_0} \alpha_k b_k^\dagger |\text{GS}\rangle$$

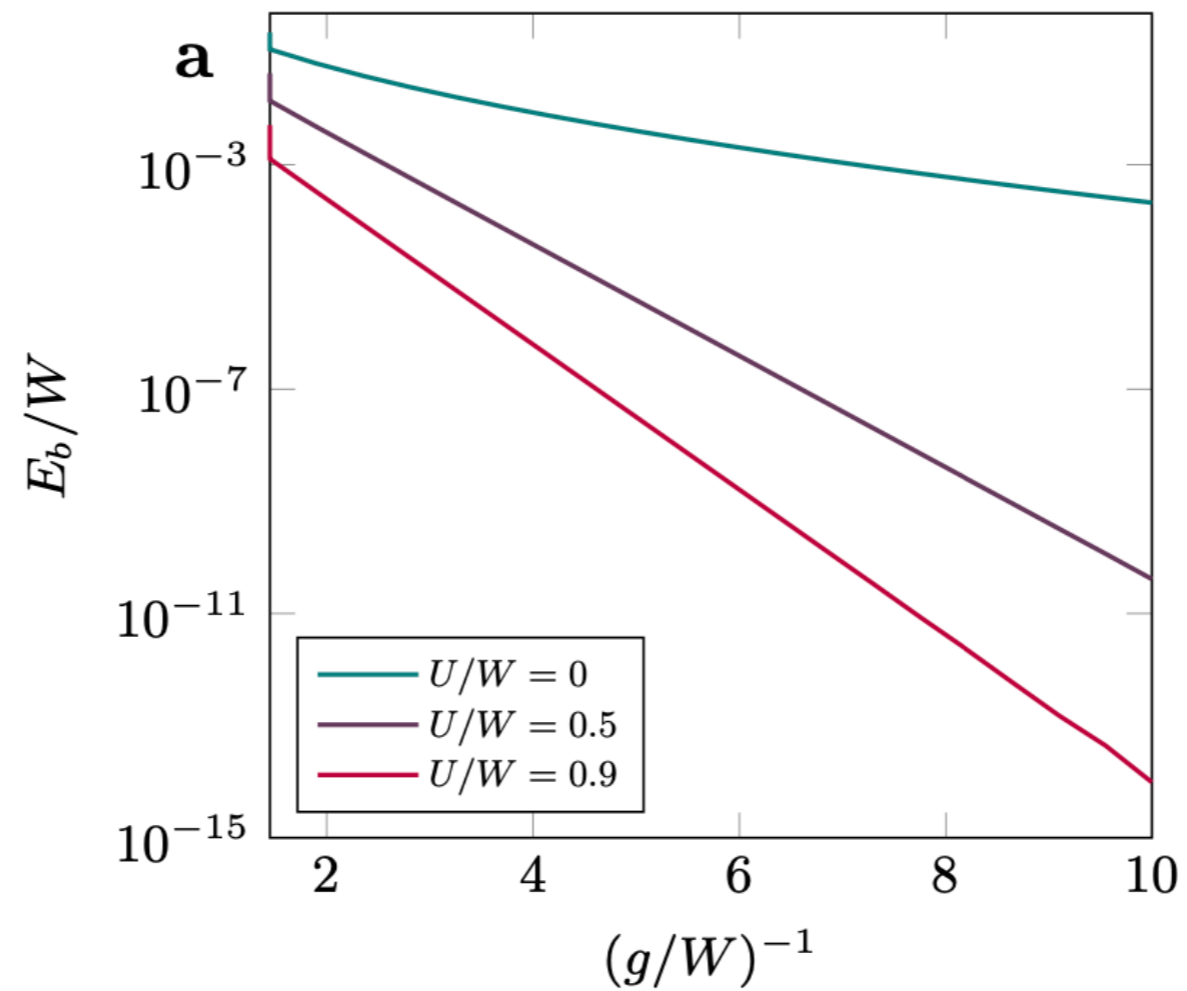


$$= \sum_{k, k'} b_k^\dagger b_{k'}$$

$$E_b = \langle \text{GS} | H | \text{GS} \rangle - \langle \psi | H | \psi \rangle \leq 0$$

Cooper Instability

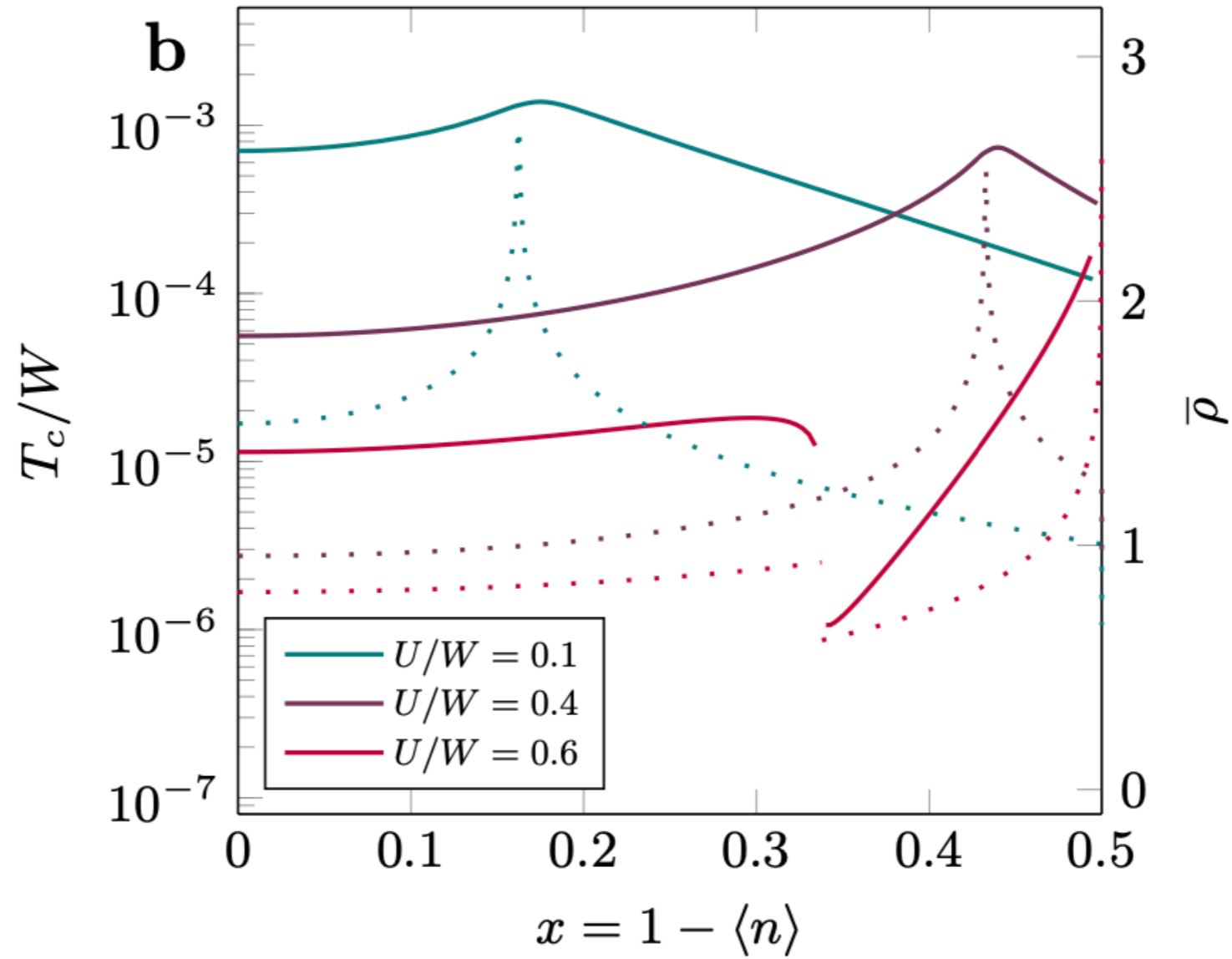
$$E_b = -E \sim W(1 - (U/W)^2)e^{-\pi W \sqrt{1 - (U/W)^2} / g}$$



Pair Susceptibility



solve for T_c



$$W > U$$

$$T_c = (W - U)^{4/5} U^{1/5} \frac{e^\gamma}{\pi} e^{-\frac{4}{5} \frac{W}{g}}.$$

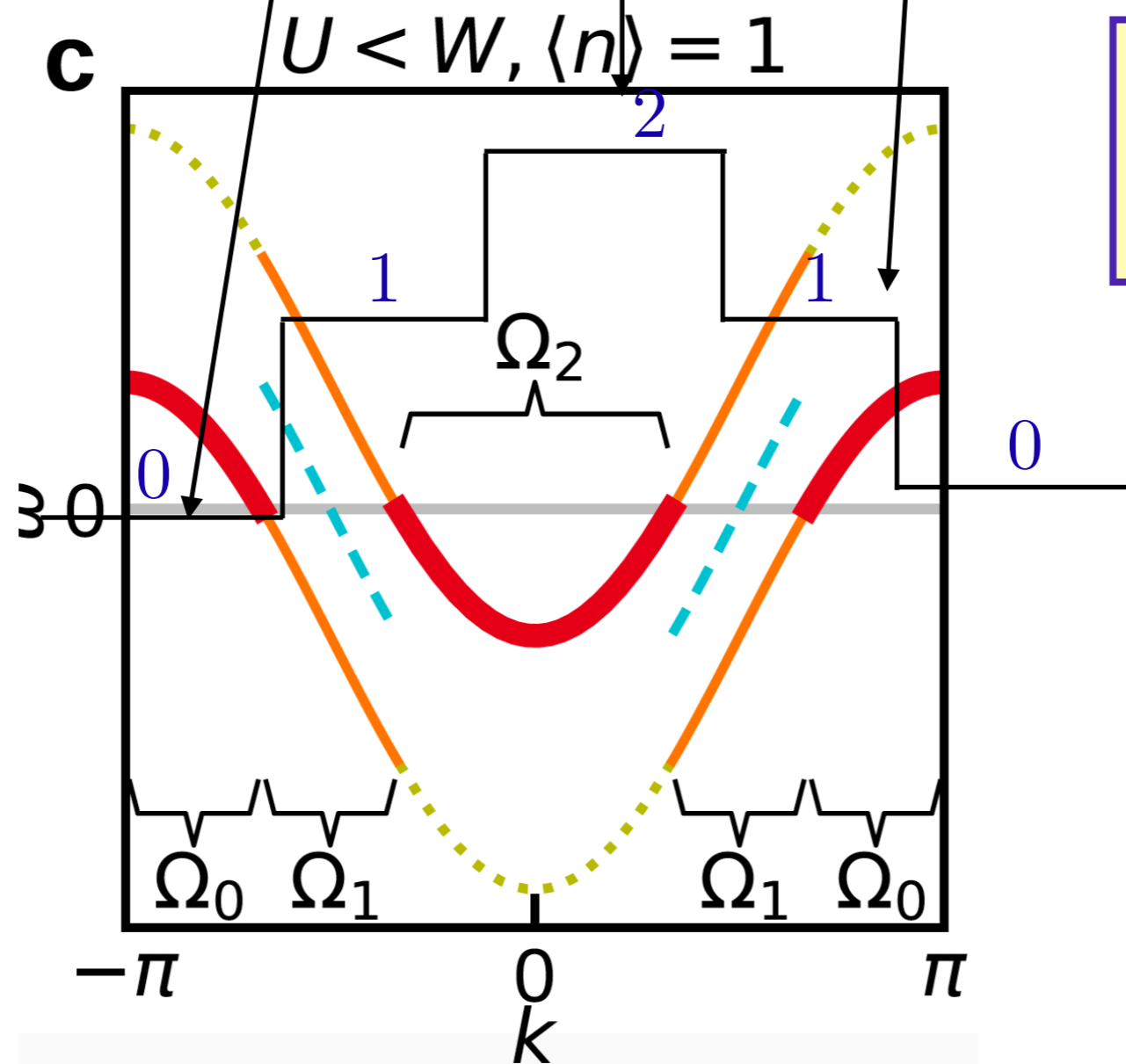
variational wave function

$$|\psi_{\text{BCS}}\rangle = \prod_k (u_k + v_k b_k^\dagger) |0\rangle$$



$$|\psi_{\text{BCS}}\rangle = \prod_{k>0} (u_k^2 + v_k^2 b_k^\dagger b_{-k}^\dagger + u_k v_k (b_k^\dagger + b_{-k}^\dagger)) |0\rangle$$

$$|\psi\rangle = \prod_{k>0} \left(x_k + y_k b_k^\dagger b_{-k}^\dagger + \frac{z_k}{\sqrt{2}} (b_k^\dagger + b_{-k}^\dagger) \right) |0\rangle$$



HK
generalization

three variational parameters

$$|x_k|^2 + |y_k|^2 + |z_k|^2 = 1$$

gap equation


$$\Delta \ll U, W$$

$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

gap/ T_c ratio

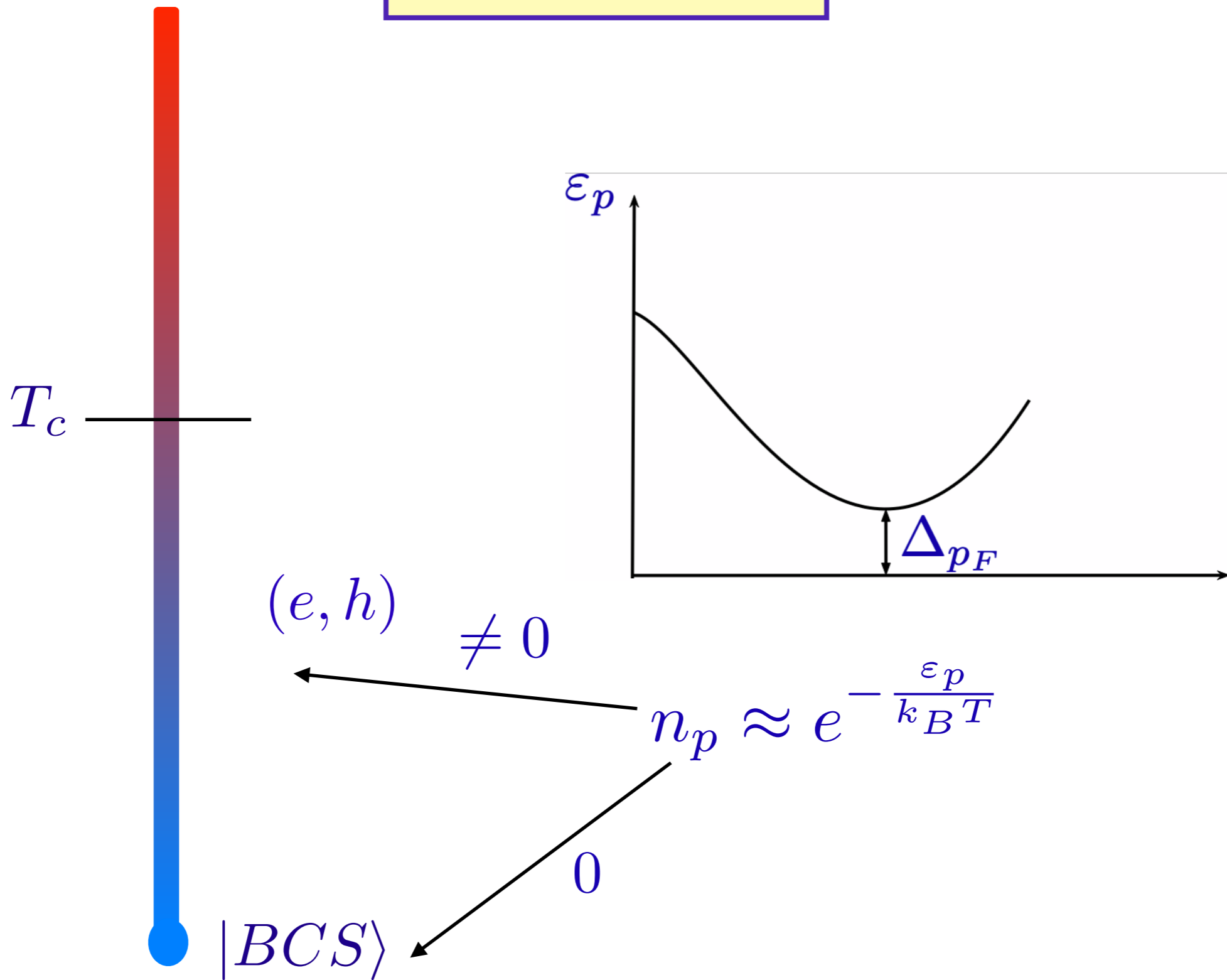
$$\Delta = (W - U)^{1/2} U^{1/2} e^{-\frac{W}{2g}}$$

$$T_c = (W - U)^{4/5} U^{1/5} \frac{e^\gamma}{\pi} e^{-\frac{4}{5} \frac{W}{g}} .$$

$$\lim_{g \rightarrow 0} \frac{\Delta}{T_c} \rightarrow \infty$$

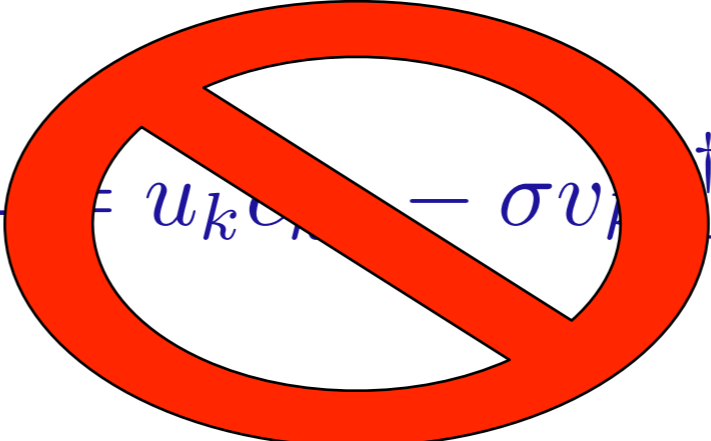
non-BCS superconductivity

excited states



Bogoliubov excitations

$$\gamma_{k\sigma} |\psi_{\text{BCS}}\rangle = 0$$

$$\gamma_{k\sigma} = u_k c_{k\sigma} - \sigma v_k c_{-k\bar{\sigma}}^\dagger$$


PYHons excitations

$$\gamma_{k\sigma}^l \propto \sqrt{2} x_k \zeta_{k\sigma}^\dagger - \sigma z_k \zeta_{-k\bar{\sigma}}$$

$$\gamma_{k\sigma}^u \propto z_k \eta_{k\sigma}^\dagger - \sigma \sqrt{2} y_k \eta_{-k\bar{\sigma}}$$

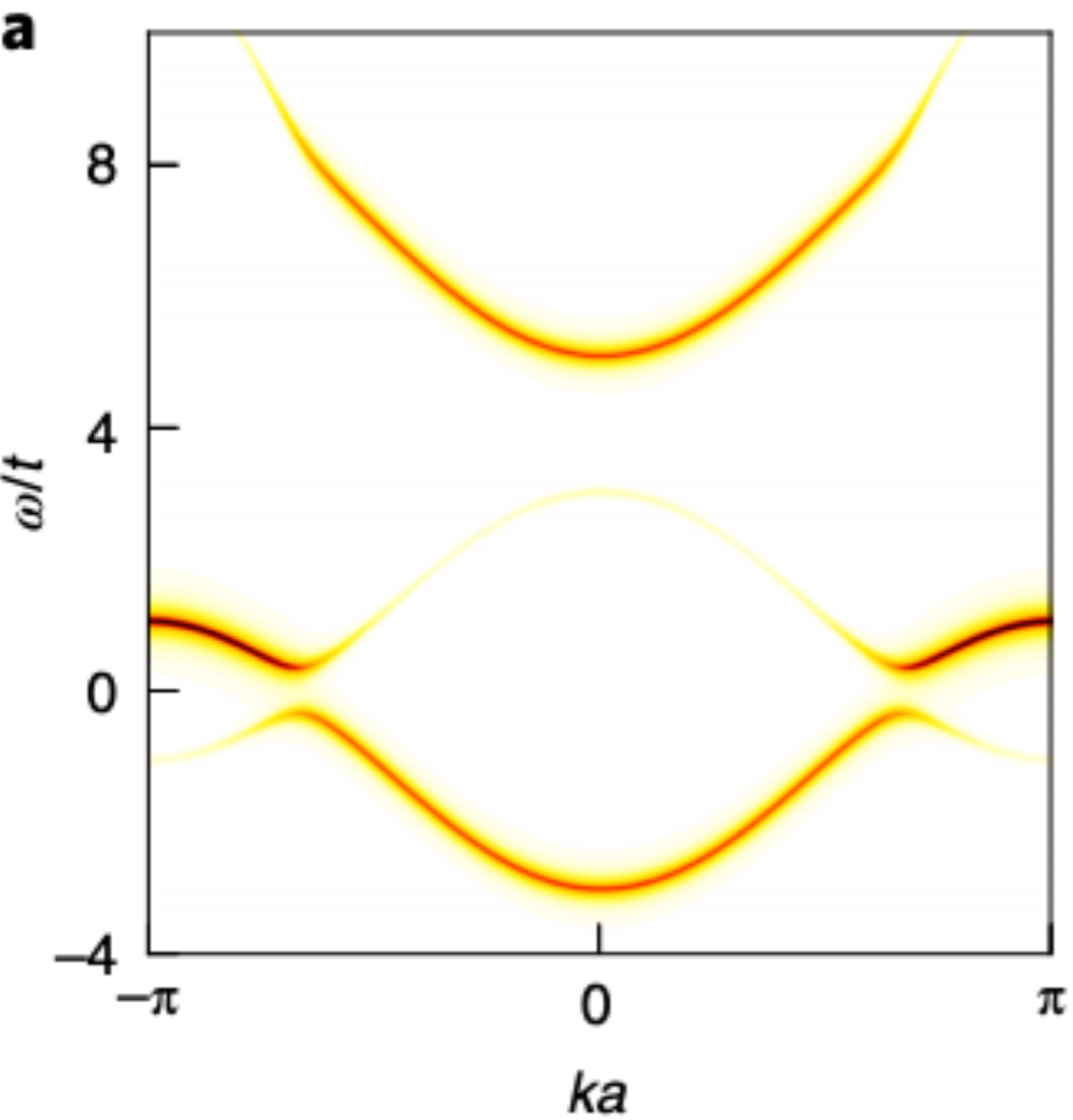
Excitation spectrum

$$\gamma_{k\sigma}^{u/l} |\psi\rangle = 0$$

$$\langle \psi | \gamma_{k\sigma}^{u/l} H \gamma_{k\sigma}^{u/l} | \psi \rangle = \langle \psi | H | \psi \rangle + E_k^{u/l}$$

$$E_k^{u/l} = \sqrt{\xi_k^{u/l^2} + \Delta^2}$$

superconductivity affects both bands!



PYHon band

can we explain the color change?

REPORT

Superconductivity-Induced Transfer of In-Plane Spectral Weight in $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$

H. J. A. Molegraaf¹, C. Presura¹, D. van der Marel^{1,*}, P. H. Kes², M. Li²

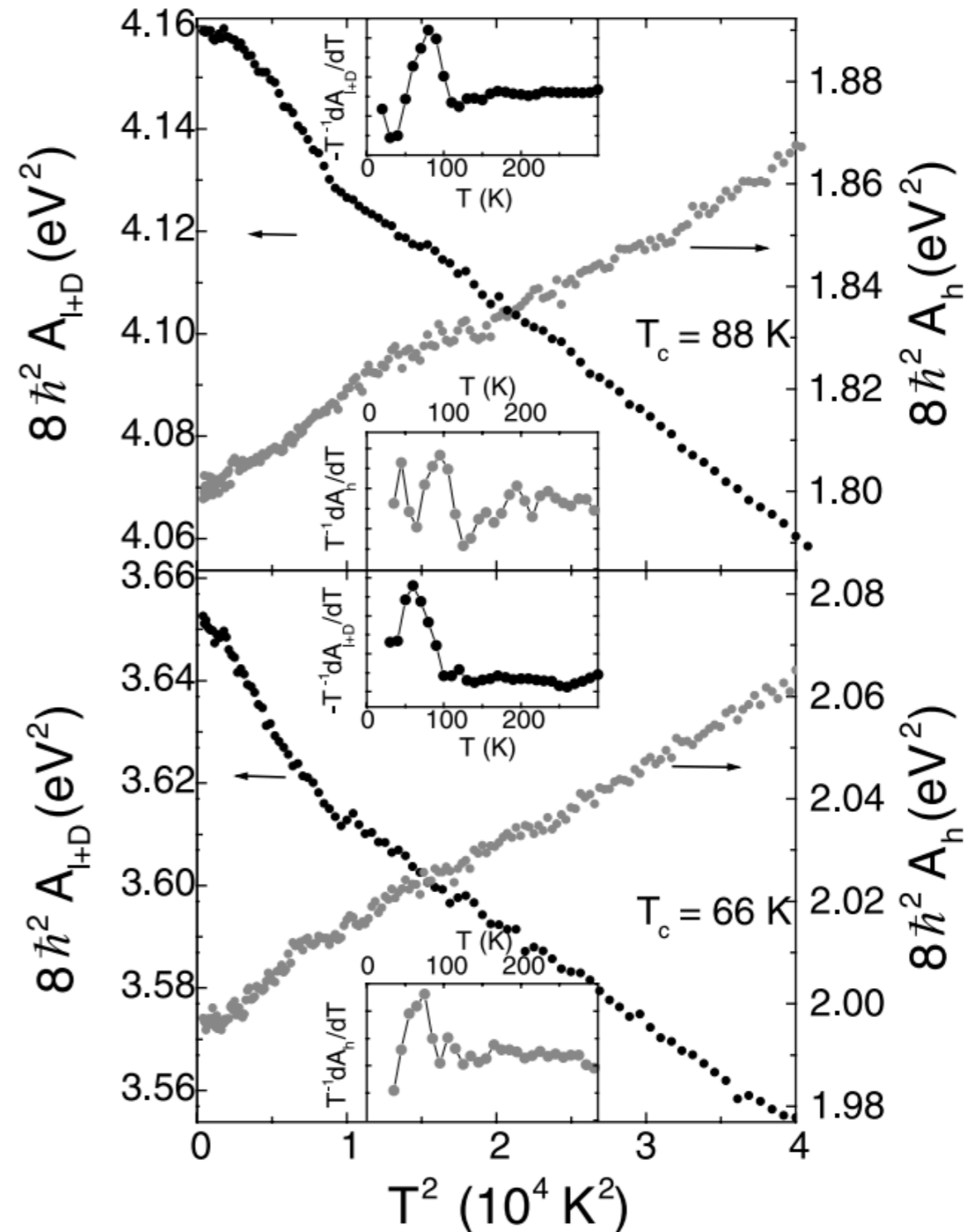
+ See all authors and affiliations

Science 22 Mar 2002:
Vol. 295, Issue 5563, pp. 2239-2241
DOI: 10.1126/science.1069947

$$A_l = \int_0^{\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 \text{ cm}^{-1}$$

$$A_h = \int_{\Omega}^{2\Omega} \sigma(\omega) d\omega \quad \Omega/2\pi c = 10000 \text{ cm}^{-1}$$

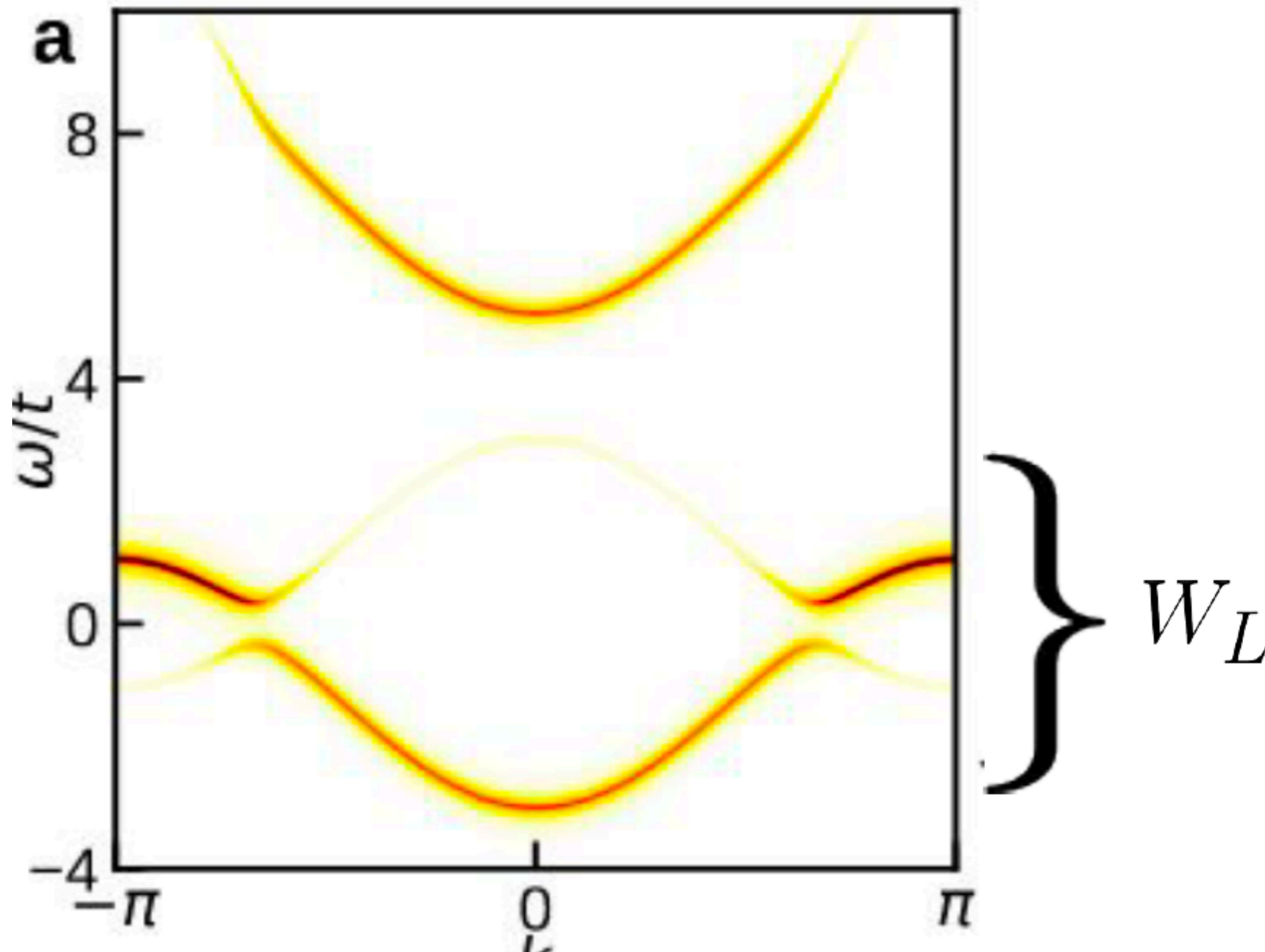
$$\frac{\Delta A_l}{A_l} \propto 3\%$$



condensation energy

Optical data are reported on a spectral weight transfer over a broad frequency range of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$, when this material became superconducting. Using spectroscopic ellipsometry, we observed the removal of a small amount of spectral weight in a broad frequency band from 10^4 cm^{-1} to at least $2 \times 10^4 \text{ cm}^{-1}$, due to the onset of superconductivity. We observed a blue shift of the *ab*-plane plasma frequency when the material became superconducting, indicating that the spectral weight was transferred to the infrared range. Our observations are in agreement with models in which superconductivity is accompanied by an increased charge carrier spectral weight. The measured spectral weight transfer is large enough to account for the condensation energy in these compounds.

UV-IR mixing



why?

$$H = H_{\text{HK}} + H_p$$

$$[H_{\text{HK}}, H_p] \neq 0$$

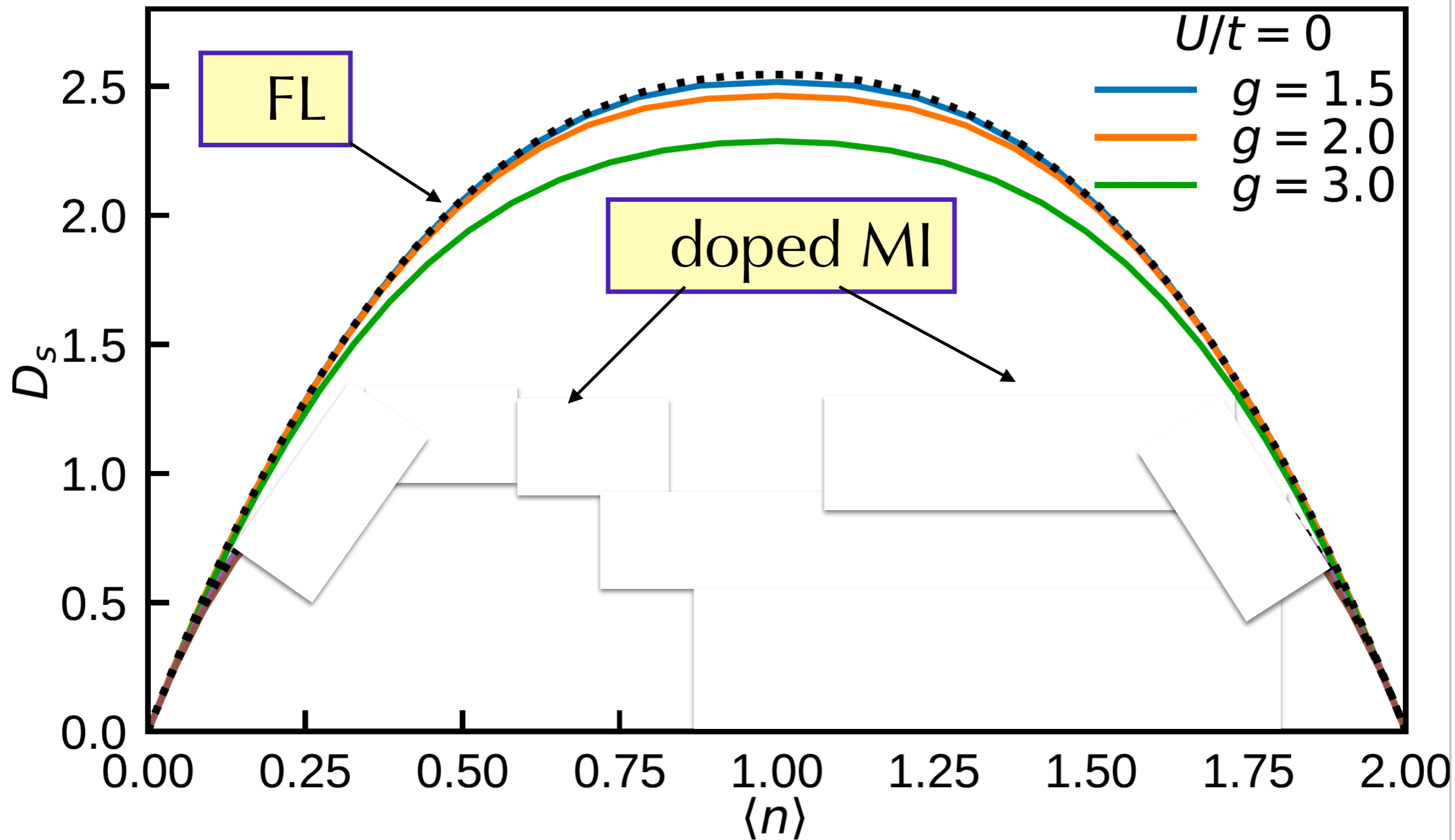


dynamical
spectral weight
transfer

is this the
general
mechanism
of the color
change?

Superfluid Density

Mottness-induced suppression



Mottness



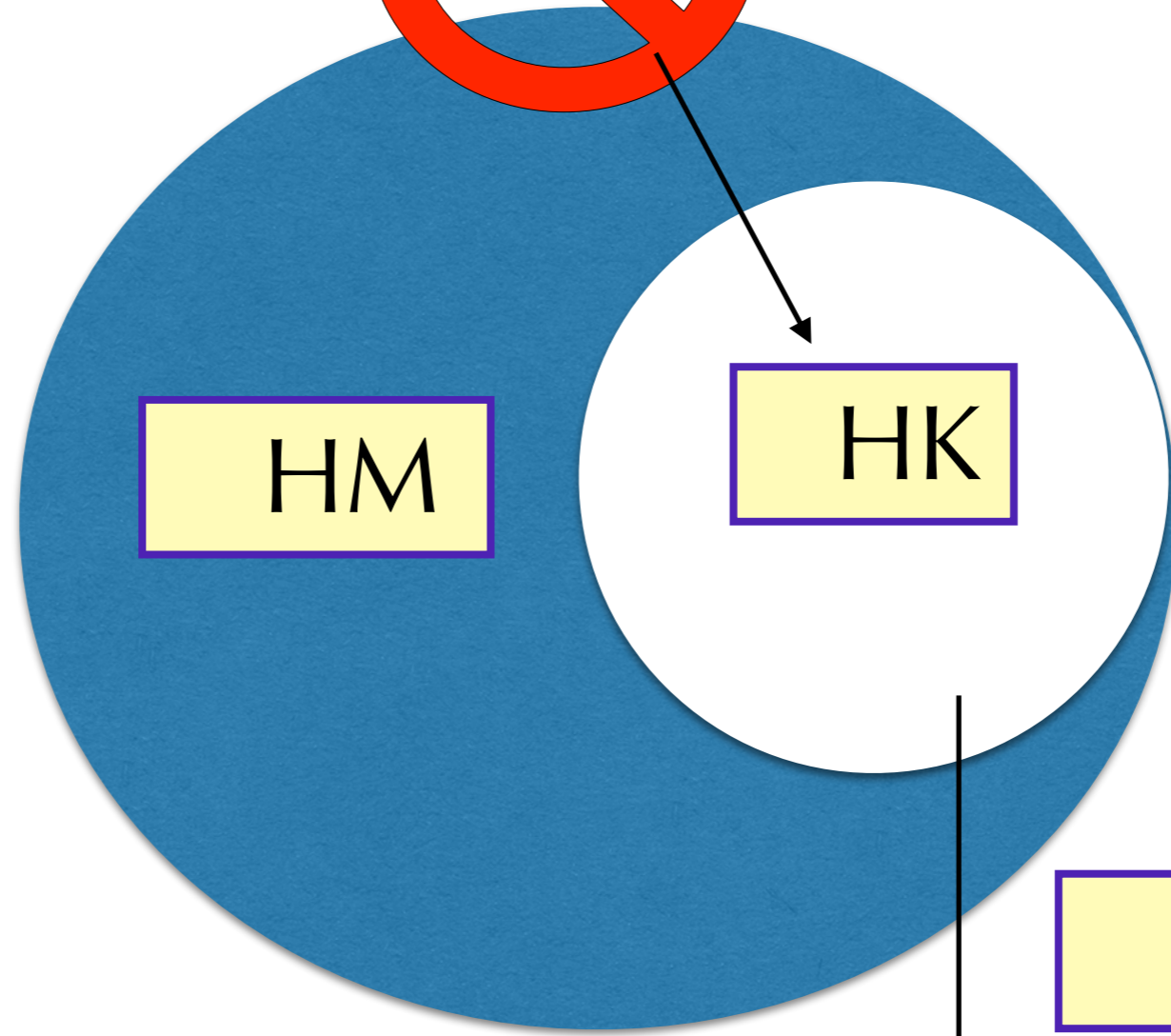
HM

HK

PYHons

violation of Luttinger

non-BCS
superconductivity





Tudor Stanescu

